1. Ms. Pham writes 2 final exams, each with 25 problems. If the exams have 12 problems in common, how many problems does she write?

   A. 24 B. 26 C. 37 D. 38 E. 49

2. A triangle has two sides of length 8.1 and 1.4. If the length of the third side is an even integer, its length must be

   A. 2 B. 4 C. 6 D. 8 E. 10

3. If (a, b) is the solution to the system of equations $\begin{cases} \pi x + (\pi + e)y = \pi + 2e \\ (\pi + 3e)x + (\pi + 4e)y = \pi + 5e \end{cases}$, find b - a.

   A. -3 B. -1 C. 0 D. 1 E. 3

4. The year 2013 has the property that when its distinct prime factors 3, 11, and 61 are each reduced by 1 and written in increasing order (that is, 2, 10, 60) each number is a factor of the next. Find the next year with this property.

   A. 2014 B. 2015 C. 2016 D. 2017 E. 2018

5. If the lines with equations y = 2x + b and y = mx - 6 intersect at a point on the x-axis, then A. mb = 12 B. mb + 12 = 0 C. m = 3b D. m + 3b = 0 E. 3m = b

6. Find the smallest positive integer value of n for which $\frac{1}{a} + \frac{1}{b} = \frac{1}{n}$ has at least three solutions (a, b) in integers with $a \geq b > 0$.

   A. 3 B. 4 C. 5 D. 6 E. 8

7. The equation $a^3 + b^2 + c^2 = 2013$ has a solution in positive integers for which b is a multiple of 5. Find a + b + c for this solution.

   A. 55 B. 57 C. 59 D. 61 E. 63

8. Each letter A through Z of the alphabet is assigned a unique integer from 2 to 27. If $A \cdot M \cdot A \cdot T \cdot Y \cdot C = 3^2 \cdot 5^2 \cdot 7 \cdot 11^2$, find M + T + Y + C.

   A. 30 B. 34 C. 36 D. 38 E. 42

9. The third-degree polynomial P(x) has only nonnegative integer coefficients. If $P(0) \cdot P(3) = 139$ and $P(1) \cdot P(2) = 689$, find P(-1).

   A. -2 B. -1 C. 0 D. 1 E. 2

10. Find the smallest positive value of t such that cos t is the same whether t is in radians or in degrees. Write your answer (rounded to 3 decimal places) in the corresponding blank on the answer sheet.

11. In quadrilateral ABCD, AB = 6, BC = 6, CD = 8, AD = 10, and $\angle C = 90°$. If the angle bisector of $\angle A$ meets diagonal BD at point E, find BE.

    A. $\frac{15}{4}$ B. 4 C. 5 D. 6 E. $\frac{25}{4}$

12. Line L has intercepts 2 and 4, while line M has intercepts 4 and 6. If L and M intersect at (a, b), which of the following could NOT be 3a + b?

A. 0 B. 4 C. 8 D. 12 E. 32

13. Sue traveled continuously starting on 1/1/2012. Her first trip was less than 3 months, and each successive trip was 2 days longer than the previous trip. If her last trip ended on 12/31/2012, which of these was the length in days of one of her trips? A. 54 B. 58 C. 65 D. 72 E. 77

14. A binary string is a sequence of 1's and 0's, such as 10011 or 11101010. How many different binary strings of length 6 are there such that no two are reversals of each other or add up to 111111?

A. 22 B. 23 C. 24 D. 25 E. 26

15. In quadrilateral PQRS, $\angle P = \angle Q = \angle S = 45°$, $\angle QPR = \angle RPS$, and $PR = 8\sqrt{2}$. Find the area of quadrilateral PQRS to the nearest integer.

A. 60 B. 61 C. 62 D. 63 E. 64

16. The numbers 2 and 1 are the smallest positive integers for which the square of the first is 2 more than twice the square of the second. If a and b are the smallest such pair with $a > 10$, find $a - b$.

A. 13 B. 15 C. 17 D. 19 E. 21

17. A number is chosen at random from among all 5-digit numbers containing exactly one each of the digits 1, 2, 3, 4, and 5. Find the probability that no two adjacent digits in the number are consecutive integers.

A. $\frac{1}{10}$ B. $\frac{7}{60}$ C. $\frac{2}{15}$ D. $\frac{3}{20}$ E. $\frac{1}{6}$

18. The triangular region with vertices (0, 0), (4, 0), and (0, 3) is rotated 90° counter-clockwise around the origin. Find the area of the figure formed by this rotation to the nearest hundredth.

A. 19.96 B. 20.04 C. 20.12 D. 20.20 E. 20.28

19. For how many pairs of positive integers (n, m) with n, m < 100 are both of the polynomials $x^2 + mx + n$ and $x^2 + mx - n$ factorable over the integers?

A. 4 B. 5 C. 6 D. 7 E. 8

20. Triangles ACD and BCD (AD = 14, BD = 40) are inscribed in a semicircle with diameter CD = 50. If AB > 25, find the area of their union.

A. 625 B. 637.5 C. 652.5 D. 673.5 E. 675

AMATYC Contest, Round 2, March 2013

(1) $25+25-12=38$ **D**

(2) $8.1-1.4<x<8.1+1.4$

$\left.\begin{array}{l}6.7<x<9.5\\ x=\text{even}\end{array}\right\}\Rightarrow x=8$ **D**

(3) $\pi x+(\pi+e)y=\pi+2e$ —

$(\pi+3e)x+(\pi+4e)y=\pi+5e$

$3ex+3ey=3e$ **E**

$x+y=1$,

$y=2,\ x=-1,\ y-x=3$

(4) $2014=2\times19\times53$,

$2015=5\times13\times31$,

$2016=2^5\times3^2\times7$,

$\boxed{1,2,6}$ **C**

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(5) $\left.\begin{array}{l}2x+b=0\to-\frac{b}{2}\\ mx-6=0\to\frac{6}{m}\end{array}\right\}\to -\frac{b}{2}=\frac{6}{m}$ **B**

$12+bm=0$

(7) $4^3+10^2+43^2=2013$ (TI)

$4+10+43=57$ **B**

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(6) $\frac{1}{a}+\frac{1}{b}=\frac{1}{n}$, $b=\frac{an}{a-n}$, $y=\frac{x\cdot4}{x-4}$ (TI)

x	5	6	8
y	20	12	8

B

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(8) $A=11$ (only!), $3+3\times5+5+7=30$ **A** (different)

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(9) $P=ax^3+bx^2+cx+d$, $P(0)\times P(3)=d(27a+9b+3c+d)=139=\text{prime}$,

$d=1$, $P(1)\times P(2)=(a+b+c+1)(8a+4b+2c+1)=689=13\times53$,

$\left.\begin{array}{l}a+b+c+1=13\\ 8a+4b+2c+1=53\\ 27a+9b+3c+1=139\end{array}\right\}\Rightarrow \begin{array}{l}a=3\\ b=5\\ c=4\end{array}$, $P=3x^3+5x^2+4x+1\Big|_{x=-1}=-1$ **B**

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(10) $\cos\frac{x\cdot\pi}{180}-\cos x=0$, $-2\cdot\sin\frac{\frac{\pi x}{180}+x}{2}\cdot\sin\frac{\frac{\pi x}{180}-x}{2}=0$, $\boxed{6.175}$

$\frac{\pi x+180x}{360}=\pi n$, $x=\frac{360\pi n}{\pi+180}$, $x_{min}=6.1754$ or $\frac{\pi x-180x}{360}=\pi n$, $x_{min}=6.3948$

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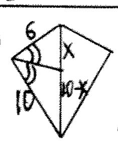
(11) 

$\frac{6}{10}=\frac{x}{10-x}$

$x=\frac{15}{4}$ **A**

(12)

(a,b)	$(-4,12)$	$(0,4)$	$(4,0)$	$(12,-4)$
$3a+b$	0	4	12	32

C

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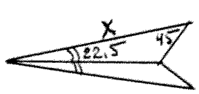
(13) $366=56+\textcircled{58}+60+62+64+66$ **B**

(15) 

$\frac{x}{\sin112.5°}=\frac{8\sqrt2}{\sin45°}$, $x=14.782$

$A=2A_1=2\cdot\frac12\cdot8\sqrt2\,x\cdot\sin22.5=64$ **E**

(14) $\text{total}=2^6=64$,

reversal of itself $=2^3=8$,

add up to 111111 : $\frac{8}{2}$ and $\frac{64-8}{2}=28$

Answer: $4+28=\boxed{32}$

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(16) $a^2=b^2\times2+2$, $a=\sqrt{2b^2+2}$ $(a,b)=(2,1);(10,7);(58,41)\ldots$ (TI) **C**

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(17)

1st digit	1 or 5	2 or 4	3	total
# of cases	2 + 2	3 + 3	4	14

$p=\frac{14}{5!}=\frac{7}{60}$ **B**

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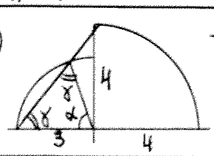
(18) 

$\tan\gamma=\frac43$, $\alpha=\pi-2\arctan\frac43$, $A_{sec}=\frac12\cdot3^2\cdot\alpha$, $A_\Delta=\frac12\cdot3^2\cdot\sin\alpha$,

$A=\frac14\pi\cdot4^2+\frac{3\cdot4}{2}+A_{sec}-A_\Delta=20.03|788059\approx20.04$ **B**

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(19) $(m,n)=(5,6);(13,30);(10,24);(15,54);(17,60);(20,96);(25,84)$ (TI) **D**

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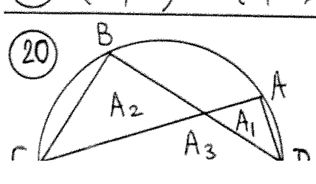
(20) 

$AC=48$, $BC=30$, $\frac{A_2}{A_1}=\left(\frac{30}{14}\right)^2\Rightarrow A_2=\frac{225}{49}A_1$ **D**

$A_3=\frac{30\times40}{2}-\frac{225}{49}A_1=\frac{48\times14}{2}-A_1\Rightarrow A_1=73.5$; $600+73.5$