1. After Ed eats 20% of a pie and Anh eats 40% of a pie, Ed has twice as much pie left as Anh. Find Ed's original amount of pie as a percentage of Anh's original amount.

A. 120 B. 125 C. 140 D. 150 E. 160

2. The expression $a\#b = ab^2 + a$ for integers $a, b > 0$. If $(a\#b)\#3 = 250$, find $a + b$.

A. 6 B. 7 C. 8 D. 9 E. 10

3. Alicia always climbs steps 1, 2, or 4 at a time. For example, she climbs 4 steps by 1-1-1-1, 1-1-2, 1-2-1, 2-1-1, 2-2, or 4. In how many ways can she climb 10 steps?

A. 81 B. 120 C. 144 D. 150 E. 169

4. The sum of six consecutive positive integers beginning at n is a perfect cube. The smallest such n is 2. Find the sum of the next two smallest such n's.

A. 679 B. 680 C. 681 D. 682 E. 683

5. The sum of the infinite geometric series S is 6, and the sum of the series whose terms are the squares of the terms of S is 15. Find the sum of the infinite geometric series with the same first term and opposite common ratio as S.

A. 2 B. 2.5 C. 3 D. 3.5 E. 4

6. When 15 is added to a set of 10 numbers, the median changes from 6 to 8. Find the median of the new set if 15 is replaced by 7.

A. 4 B. 5 C. 5.5 D. 6 E. 7

7. Rectangle SMLA has SM = 5 and ML = 10. If the two unit squares at S and M are removed, leaving 48 squares, how many of the following four sets of rectangles can exactly cover SMLA: 24 1x2s, 16 1x3s, 12 1x4s, 8 2x3s?

A. 0 B. 1 C. 2 D. 3 E. 4

8. In ΔSML, SM = 17 and ML = 12. If SL is an integer greater than SM or ML, find the smallest value of SL for which ΔSML has an obtuse angle.

A. 12 B. 20 C. 21 D. 22 E. 28

9. A polynomial with nonnegative integer coefficients has P(0) = 3, P(1) = 8, P(2) = 39, and P(3) = 144. Find P(-2).

A. -7 B. -5 C. -3 D. -2 E. -1

10. Each digit of a 10-digit number N is either a 1, 2, or 3. Every 3 consecutive digits of N form a prime number. Find the final two digits of the smallest such N.

A. 11 B. 13 C. 21 D. 23 E. 31

11. Multiplying the corresponding terms of a geometric and an arithmetic sequence yields 96, 180, 324, 567, ... Find the next term of the new sequence.

A. 960 B. 972 C. 980 D. 984 E. 988

12. If $\log_x y + \log_y x = 2.9$ and xy = 128, find x + y.

A. 32 (B.) 36 C. 40 D. 48 E. 64

13. The equation $a^5 + b^2 + c^2 = 2011$ (a, b, c positive integers) has a solution in which two of the three numbers are prime. Find the value of the nonprime number.

A. 38 B. 40 (C.) 42 D. 44 E. 46

14. A palindrome is a number like 121 or 1551 which reads the same from right to left and from left to right. How many 4-digit palindromes are divisible by 17?

A. 2 B. 4 (C.) 5 D. 6 E. 8

15. Six numbers are selected from 0, 1, ..., 6 and arranged in a 2x3 grid so that each row is increasing from left to right and each column is increasing from top to bottom. Find the number of such different arrangements.

(A.) 24 B. 28 C. 30 D. 35 E. 42

16. The increasing sequence of positive integers $a_1, a_2, a_3, \cdots$ satisfies the equation $a_{n+2} = a_n + a_{n+1}$ for all $n \geq 1$. If $a_7 = 160$, find a_8.

A. 257 B. 258 (C.) 259 D. 260 E. 261

17. For how many integers $1 \leq n \leq 2011$ is the fraction $\frac{n^2+7}{n+4}$ NOT in lowest terms?

A. 85 B. 86 (C.) 87 D. 88 E. 89

18. Ten sets of coins each contain one penny, and the kth set has $2k$ dimes for $1 \leq k \leq 10$. If one coin is selected at random from each set, find the probability that the number of pennies in the selection is odd.

(A.) 10/21 B. 11/23 C. 1/2 D. 11/21 E. 12/23

19. Every set {1, 2, 3, ..., n} can be split into sets so that each set sums to the same total. For example, {1, ..., 7} = { 1, 2, 4, 7} ∪ {3, 5, 6}; each set sums to 14. Find the largest number of such equal sum sets into which {1, 2, 3, ..., 15} can be split.

A. 4 B. 5 C. 6 (D.) 8 E. 10

20. If the nine integers 2 through 10 are arranged at random in a row, find the probability that no two prime numbers are next to each other.

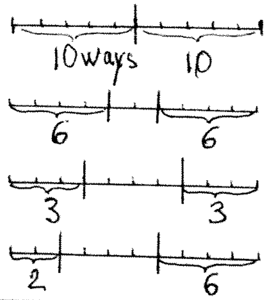
A. $\frac{1}{14}$ B. $\frac{2}{21}$ (C.) $\frac{5}{42}$ D. $\frac{1}{7}$ E. $\frac{1}{6}$

AMATYC Contest, Feb—Mar 2011

① Ed — $0.8x$, Anh — $0.6y$, $0.8x = 2\times 0.6y$, $x = 1.5y$ (D)

② $(a\#b)\#3 = (a\#b)(3^2+1) = a(b^2+1)\cdot 10 = 250$, $a=5$, $b=2$ (B)

③ a) b) c) d)



a) 10×10
b) $+\,6\times 6$
c) $+\,3\times 3$
d) $+\,(2\times 6)\cdot 2 = 169$ (E)

④ $n+(n+1)+\dots+(n+5) = x^3$, $6n+15 = x^3$, $n = \frac{x^3-15}{6}$, $n = 2, 119, 560, \dots$ (A)

⑤ $S_1 = \frac{a}{1-r} = 6$, $S_2 = \frac{a^2}{1-r^2} = 15$, $\frac{a}{1-r}\cdot\frac{a}{1+r} = \frac{a^2}{1-r^2}$, $6S_3 = 15$, $S_3 = 2.5$ (B)

⑥ median $= \frac{a+b}{2} = 6$, $b=8$, $a=4$, 7 is between 4 and 8 (E)

⑦ only set 16 1×3s (B)

⑧ $\sqrt{12^2+17^2} \approx 20.8$, $\min(SL) > 20.8$ (C)

⑨ $3^5 > 144 \Rightarrow P = ax^4+bx^3+cx^2+dx+3$

if $a=1$,

$$\left.\begin{aligned} b+c+d &= 4 \\ 8b+4c+2d &= 20 \\ 27b+9c+3d &= 60 \end{aligned}\right\} \Rightarrow \begin{aligned} &b=d=2,\ c=0 \\ &P(x) = x^4+2x^3+2x+3 \\ &P(2) = -1 \end{aligned}$$

(E)

⑩ 113113113 1131 — 113 1131131

the last two digits 31 (E)

⑪ $a, ar, ar^2, ar^3, ar^4 \dots$ $\quad ab = 96$, $abr + adr = 180 \longrightarrow r(96+ad) = 180$

$b, b+d, b+2d, b+3d, b+4d \dots$ $\quad abr^2 + 2adr^2 = 324$ $\quad r^2(96+2ad) = 324$

$96r^2 = 360r - 324$, $r = \frac{3}{2}, \frac{9}{4}$, $ad = 24$, $abr^4 + 4adr^4 = 972$ (B)

⑫ $z = \log_x y$, $z + \frac{1}{z} = 2.9 \Rightarrow z = \frac{2}{5}, \frac{5}{2}$, $\begin{cases} x^5 = y^2 \\ xy = 2^7 \end{cases} \Rightarrow \begin{matrix} y = 2^5 \\ x = 2^2 \end{matrix}$, $x+y = 36$ (B)

⑬ if $a=3$, $b = \sqrt{1768-c^2}$, $c=2$, $b=42$ (graphing calculator) (C)

⑭ 5 numbers: 2992, 3553, 4114, 7667, 8228 (C)

⑮
1	3	5
2	4	6

or

1	2	5
3	4	6

or

1	2	3
4	5	6

Total 5×7 (D)

⑯ $a_7 = 5a_1 + 8a_2 = 160$, $5a_1 = 40$, $8a_2 = 120$, $a_8 = 8a_1 + 13a_2$ (C)

⑰ $\frac{n^2+7}{n+4} = \frac{n^2+4n}{n+4} + \frac{-4n-16}{n+4} + \frac{23}{n+4} \Rightarrow n = 19+23K \le 2011$, $K = 0, 1, \dots, 86$ (C)

⑱ by mathematical induction, for one set $\frac{1¢}{10¢}\Big|\frac{1}{2}$ $P_1(\text{odd}) = \frac{1}{3} = \frac{1}{2\cdot 1+1}$;

1¢	1	1	1 …	1	1
10¢	2	4	6 …	2K	2K+2

suppose for $n=k$, $P_k(\text{odd}) = \frac{K}{2K+1}$, and show (A)

$P_{K+1}(\text{odd}) = \frac{K+1}{2(K+1)+1}$; $P_{K+1}(\text{odd}) = \frac{K}{2K+1}\cdot\frac{2K+2}{2K+3} + \left(1 - \frac{K}{2K+1}\right)\cdot\frac{1}{2K+3} = \frac{K+1}{2K+3}$

⑲ 8 equal sum sets: {15}, {14, 1}, {13, 2}, {12, 3}, {11, 4}, {10, 5}, {9, 6}, {8, 7} (D)

⑳ if the first 2 primes on 1st & 5th — 1 way, on 1st & 4th — 3, on 1st & 3rd — 6, on 2nd & 5th — 1, 2nd & 4th — 3, 3rd & 5th — 1, total = 15, $p = \frac{15}{C(9,4)} = \frac{5}{42}$ (C)