

Tuning of a Numerical Integration System

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Abstract

The simulation of planetesimals in the presence of Jovian planets is carried out by a numerical integration system. It composes of a 4th order Runge-Kutta integrator with Richardson extrapolation, and a 13th order Störmer integrator. Such an integrator has many uses beyond its intended application. Work is done to generalize the system so that it can be extended to perform other simulations, such as meteor stream modeling.

Background

Dynamic simulations have been performed using a high-accuracy and stable integration system by making use of a modified 13th order Störmer method. It is implemented by Kevin Grazier and employs a few computational techniques to improve its accuracy (1). The error growth of this integrator approaches the limits as specified by Brouwers's Theorem (2). In order to compute the values of the next time step, the Störmer method requires parameters for the previous 13 states. A 4th order Runge-Kutta integrator with Richardson extrapolation (RK4R) is used to generate these values to start the integrator. The initial values are calculated in quadruple precision then converted into double precision numbers. This ensures the values passed to the Störmer integrator are as accurate as possible.

The first application of this system was used to simulate planetesimals in the presence of Jovian planets, and study the evolution of these particles over the time span of about one billion years. Particles remaining after this duration can be considered fairly stable. This simulation ignores the inner planets to simplify the model. The distance of the inner planets from the Jovian planets is large so we can consider gravitational pull originating from a point source. This would also speed up the simulation time significantly.

Currently the integration system is being analyzed and reworked to perform meteor stream simulations in the inner solar system. Simulations in the inner solar system require that all the planets be simulated. Relativistic corrections would also need to be implemented in order to simulate Mercury correctly, as it cannot be accurately simulated with just Newtonian mechanics. Some of this work has been done by previous CURE

student, John Palombi, but using an older codebase. So the goal is to resolve the differences, merge, and verify the code so that two different versions do not need to be maintained.

Methodology

To implement relativistic corrections for Mercury, the acceleration of a particle/planet is computed normally (N-body problem with Newton's law of universal gravitation), and then the contribution to the acceleration due to relativity is added to the acceleration. It is characterized by the following pseudo code, though (1:3) is language-specific for vectors in FORTRAN.

$$\begin{aligned}
 &\text{For each planet } i \\
 &v(1:3) = \text{velocity}(Mercury) \\
 &n(1:3) = \frac{r(i, 1:3)}{dr(i, 1:3)} \\
 &vtr_1(1:3) = 4.0 \left(\frac{\mu_0}{c\text{speed}} \right)^2 \cdot \frac{n(1:3)}{dr(i, 1:3)^3} \\
 &scl_1 = \frac{\mu_0}{(c\text{speed} \cdot dr(i, 1:3))^2} \\
 &scl_2 = v(1:3) \cdot n(1:3) \\
 &scl_3 = v(1:3) \cdot v(1:3) \\
 &vtr_2(1:3) = 4.0 \cdot scl_2 \cdot v(1:3) - scl_3 \cdot n(1:3) \\
 &acc(i, 1:3) = acc(i, 1:3) + vtr_1(1:3) + scl_1 \cdot vtr_2(1:3)
 \end{aligned}$$

where $r(i, 1:3)$ indicates the x, y, z position of the i th body, $dr(i, 1:3)$ being the displacement vector from Mercury to the i th body, $\mu_0 = GM_{Mercury}$, $c\text{speed}$ is the speed of light in AU/day , and $acc(i, 1:3)$ being the acceleration vector.

Progress

The differences between the code bases have been identified, and most of the code have been merged. The relativity computation has been re-implemented and manually debugged. The RK4R integrator and the collision detection algorithm (for close encounters) have also been manually debugged, and no problems have been found. So, the discrepancy demonstrated by the visualization module appears to be a weakness in the RK4R integrator itself, and not an error in its implementation (3). Replacing the RK4R initial solutions generator with the Bulirsch-Stoer algorithm has been considered, but the possible advantages it has does not outweigh the work and planning involved in changing it. The

definitive status of the integrator is unknown as there was no access to the computing servers to run simulation test-runs.

As of last year, the code has remained in the verification stage, and unfortunately still remains here. There has not been significant progress in verifying the code or changing/adding core functionality; this integration system is suffering from the classic case of software aging. It has been written using old paradigms, and before computational power reached an acceptable threshold to support simulations of this nature. Many programming tricks are used to improve the performance, sometimes at the cost of readability. As a result, the integration code is difficult to debug as it makes use of several global variables (which increase performance vs. local variables), which results in long dependency chains. This means unit testing requires identifying the interactions between functions and accounting for them.

Future Work

Initial mathematics and planning has been done to simulate the Moon. The Moon would be simulated differently, as adding the Moon to be computed by the Störmer method would severely compromise the performance of the integration system. This is because the integrator requires the period of the step size, $\Delta h = \frac{period}{nsteps=1024}$, to be the particle with the lowest frequency. Therefore, its orbit would be crudely computed using Kepler's equation with Newton's method in a two-body fashion. The gravitational contribution by the Moon would be small, so accuracy would not suffer very much. Future work would finish the implementation of this.

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